by the cross section where the flow rate vanishes, which in the case of sources corresponds to $X > 2 / N$ and in that of sinks to $-X < 2 / N$.

REFERENCES

1. Berman, A. S., Laminar flow in channels with porous walls. J. Appl. Phys., Vol. 24, № 9, 1953.
2. Terrill, R. M. and Thomas, P. W., On laminar flow through a uniformly porous pipe. Appl. Scient. Res., Vol. 21, № 1, 1969.

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ON THE PROBLEM OF DESALINIZATION OF SOIL CONTAINING READILY SOLUBLE SALTS

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A general solution of the problem of desalinization of soil containing rapidly soluble salts is given. The salts are initially nonuniformly distributed, and it is assumed that they pass instantaneously from the solid phase to the solution. A condition of the third kind is postulated at the soil surface, reflecting the continuity of the mass flux of the salts.

A different approach was used in [1] to construct a solution for a particular case of homogeneous salinization, and the problem of uniqueness of the solution was studied. The process of diffusion of salts in the course of washing the soil was also studied in [2].

The mathematical formulation of the problem has the form:

$$\begin{gathered} Dc_{\xi\xi} - vc_\xi = mc_\tau, \quad 0 < \xi < \xi_0(\tau) \\ -Dc_\xi + vc = vc_n, \quad \xi = 0 \\ Dc_\xi = \varphi(\xi)\, d\xi_0(\tau) / d\tau, \quad \xi = \xi_0(\tau) \end{gathered} \tag{1}$$

Here the diffusion coefficient D, rate of filtration v, porosity m and concentration c_n of the wash water are all assumed constant; $\xi_0(\tau) = v\tau / m$ denotes the front of the flow of water, ξ is the coordinate counted from the soil surface, τ is time, $c(\xi, \tau)$ is the concentration of the solution in motion and $\varphi(\xi)$ is an arbitrarily prescribed function of the initial bulk salinity of the soil. The latter function is subjected to the usual constraints imposed on the original of a Laplace transform.

Introducing the dimensionless variables x, t and the functions $u(x, t)$, we can reduce the problem (1) to the form

$$u_{xx} = u_t, \quad 0 < x < t, \; u_x - u / 2 = 0, \quad x = 0 \tag{2}$$

$$u_x + u / 2 = f(t) \exp(-t / 4), \quad x = t \tag{3}$$

$$x = \frac{v\xi}{D}, \quad t = \frac{v^2\tau}{mD}, \quad f(t) = \frac{1}{m}\varphi\left(\frac{Dt}{v}\right)$$

$$u(x, t) = [c(x, t) - c_n] \exp\left(\frac{t}{4} - \frac{x}{2}\right)$$

The heat conductivity equation (2) has a one-parameter family of solutions

$$W(x, t; \alpha) = \exp(-\alpha^2 t)\,[A(\alpha) \sin \alpha x + B(\alpha) \cos \alpha x]$$

When $B = 2\alpha A(\alpha)$, the function $W(x, t; \alpha)$ satisfies the first condition of (3). Let us write the solution $u(x, t)$ of the problem (2),(3) in the form

$$u(x, t) = \int_0^\infty A(\alpha)(\sin \alpha x + 2\alpha \cos \alpha x) \exp(-\alpha^2 t)\, d\alpha \qquad (4)$$

Taking into account the boundary condition at $x = t$, we obtain the following integral equation for the function $A(\alpha)$:

$$\int_0^\infty A(\alpha)\left[\left(\frac{1}{4} - \alpha^2\right) \sin \alpha t + \alpha \cos \alpha t\right] \exp\left[\left(\frac{1}{4} - \alpha^2\right) t\right] d\alpha = \frac{1}{2} f(t) \qquad (5)$$

Let $A(\alpha) \in L(0, \infty)$ and $A(0) = 0$. Then, integrating both parts of (5) in t from 0 to t and changing the order of integration in the left-hand side (which can be done by virtue of the uniform convergence in t), we obtain

$$\int_0^\infty A(\alpha) \sin \alpha t \exp\left[\left(\frac{1}{4} - \alpha^2\right) t\right] d\alpha = \frac{1}{2} F(t) \qquad (6)$$

$$\left(F(t) = \int_0^t f(z)\, dz\right)$$

We denote by

$$L_p(h) \equiv \int_0^\infty e^{-ps} h(s)\, ds = H(p), \quad L_s^{-1}(H) \equiv \frac{1}{2\pi i} \int_{d-i\infty}^{d+i\infty} e^{ps} H(p)\, dp = h(s)$$

the forward and inverse Laplace transform, respectively.

Applying the operation L_{p^2} to both sides of Eq. (6) and changing the order of integration which is allowed by the properties of the function $A(\alpha)$ given above, we obtain

$$\int_0^\infty A(\alpha) \frac{\alpha\, d\alpha}{\alpha^2 + (p^2 + \alpha^2 - 1/4)^2} = \frac{1}{2} L_{p^2}(F) \qquad (7)$$

We can easily see that

$$\frac{\alpha p}{\alpha^2 + (p^2 + \alpha^2 - 1/4)^2} = \frac{\alpha p}{(p^2 + \alpha^2 + 1/4)^2 - p^2} = \frac{1}{2}\left[\frac{\alpha}{(p - 1/2)^2 + \alpha^2} - \frac{\alpha}{(p + 1/2)^2 + \alpha^2}\right]$$

therefore, multiplying both sides of (7) by p and operating with L_s^{-1}, we find that

$$\frac{1}{2} L_s^{-1}[p L_{p^2}(F)] = \operatorname{sh} \frac{s}{2} \int_0^\infty A(\alpha) \sin \alpha s\, d\alpha$$

Thus the function

$$\psi(s) = \frac{1}{2} \operatorname{sh}^{-1} \frac{s}{2} L_s^{-1}[p L_{p^2}(F)] = \frac{1}{2} \operatorname{sh}^{-1} \frac{s}{2} L_s^{-1}[p^{-1} L_{p^2}(f)]$$

is a sine transform of $A(\alpha)$. The latter is recovered using the following transformation formula [3]:

$$A(\alpha) = \frac{2}{\pi} \int_0^\infty \psi(s) \sin s\alpha \, ds \tag{8}$$

and it satisfies the assumptions made above.

We can transform the expression (4) to the form suitable for numerical computations using the known formula (see [4])

$$\int_0^\infty \exp(-\alpha^2 t) \sin s\alpha \sin x\alpha \, d\alpha = \frac{\pi}{2} G(x, s; t) \tag{9}$$

$$G(x, s; t) = \frac{1}{2\sqrt{\pi t}} \left\{ \exp\left[-\frac{(s-x)^2}{4t} \right] - \exp\left[-\frac{(s+x)^2}{4t} \right] \right\}$$

Substituting (8) into (4), changing the order of integration and taking due account of (9), we obtain

$$u(x, t) = \int_0^\infty \psi(s) \left[G(x, s; t) + \frac{\partial G}{\partial x}(x, s; t) \right] ds = \tag{10}$$

$$\frac{1}{2\sqrt{\pi t}} \int_0^\infty \psi(s) \left\{ \left(\frac{s-x}{t} + 1 \right) \exp\left[-\frac{(s-x)^2}{4t} \right] + \right.$$

$$\left. \left(\frac{s+x}{t} - 1 \right) \exp\left[-\frac{(s+x)^2}{4t} \right] \right\} ds$$

It can be confirmed by direct substitution that the function $u(x, t)$ given by (10) satisfies all conditions of the problem (2),(3).

Consider an example. In the course of salinization, the crystals of the readily soluble salts accumulate, in most cases, near the daylight surface of the soil. We can therefore take $f = \exp(-\mu^2 t)$ where $\mu = \text{const}$. We then have, successively,

$$L_{p^2}(f) = (p^2 + \mu^2)^{-1}, \quad p^{-1} L_{p^2}(f) = [p(p^2 + \mu^2)]^{-1}$$

$$L_s^{-1}[p^{-1} L_{p^2}(f)] = \frac{1 - \cos \mu s}{\mu^2}, \quad \psi(s) = \frac{1 - \cos \mu s}{2\mu^2 \operatorname{sh}(s/2)}$$

REFERENCES

1. Kapranov, Iu. I., On certain exact solutions in the problems of salinization of soil. Izv. Akad. Nauk. SSSR, MZhG, № 1, 1972.
2. Pen'kovskii, V. I., On the problem of mathematical simulation of the process of soil desalinization. PMTF, № 5, 1975.
3. Ditkin, V. A. and Prudnikov, A. P., Integral Transforms and Operational Calculus. (English translation), Pergamon Press, Book № 10044, 1966.
4. Gradshtein, I. S. and Ryzhik, I. M., Tables of Integrals, Sums, Series and Products. Moscow, Fizmatgiz, 1962.

Translated by L. K.